Lecture no.2

Basic Concepts about Nonideal Plasma

"Charge-Atom" Interactions in Nonideal Plasma

In the previous lecture we have considered interaction models between charged particles only. Notice that in partially ionized plasma (at high densities and low temperatures) the interactions between charged and neutral particles are dominant and cannot be described in an ideal–gas approximation. The effects of nonideality due to charge–neutral interaction are important for such properties as the electrical conductivity, thermal conductivity, thermoelectromotive force etc.

The interaction between isolated classical atoms and charges. Let us consider the potential produced by atoms (molecules) at the ion (electron) location. It is known that the full ion–atom (electron–atom) interaction consists of the exchange, electric, and polarization interactions. The polarization force is appreciable even at low densities due to the its long range character. An atom polarized by microfields of ions (electrons) creates at the ion (electron) location the polarization potential, thus we obtain the following polarization interaction potential for isolated classical atoms and ions (electrons):

$$\varphi(r) = -\frac{\alpha e^2}{2r^4} \quad , \qquad (1)$$

where $r > r_a$ is the distance between particles; α is the polarizability of the atom and r_a is the atomic radius. The total potential is created by all atoms at the ion location is:

$$\varphi = 4\pi \int_{r_a}^{\infty} \varphi(r) n_a(r) r^2 dr \quad , \qquad (2)$$

here $n_a(r)$ is the atomic number density which depends on the distance from the ion. If the ion-atom interaction is not so strong the dependence $n_a(r)$ can be neglected, i.e., $n_a \neq n_a(r)$. Then

$$\varphi = -2\pi e^2 n_a \alpha / r_a \ . \qquad (3)$$

The ideality criterion can be written in the following form:

$$\Gamma_{ia} = \frac{2\pi e^2 n_a \alpha}{r_a k_B T} \ll 1 \ , \qquad (4)$$

where r_a is the "cut-off" radius of the polarization interaction (this radius is approximately equal to the atomic linear size). It should be noted that the effects of nonideality caused by charge–neutral interaction can occur in highly polarizable gases such as metal vapors. For instance, in the case of cesium plasma $\alpha = 400 a_B^3$; $r_a = 4a_B$ and at $T = 2000\,K$ we have $\Gamma_{ia} \le 0{,}1$ as long as $n_a < 10^{19}\,cm^{-3}$.

Notice that the electron–atom interaction potential has the same polarization asymptote $\varphi(r)$ but it cannot be adequately determined in wide range of temperatures because in this case we have to calculate the electron-atom scattering phases. It is the separate and complicated problem. However, the electron–atom interaction at low temperatures can be described by a single parameter, namely, the electron–atom scattering length L (as long as $n_a |L|^3 \ll 1$). Then the real potential $\Phi(r)$ can be replaced by a δ-like potential:

$$\Phi(r) = 2\pi h^2 L \delta(r) / m \ . \qquad (5)$$

The electron–atom interaction energy is calculated by following relation:

$$U = \int n_a(\vec{r}')\Phi(\vec{r}-\vec{r}')\left|\Psi(\vec{r})\right|^2 d\vec{r}d\vec{r}' \ , \qquad (6)$$

where $\Psi(\vec{r})$ is the electron wavefunction. For weakly coupled plasma the electron–atom correlations can be neglected and using Eq. (5) we obtain:

$$U = 2\pi h^2 L n_a / m \ . \qquad (7)$$

Finally, the criterion of plasma ideality corresponding to the electron–atom interaction has the following form:

$$\Gamma_{ea} = \frac{2\pi |L| h^2 n_a}{m k_B T} \ll 1 \quad . \tag{8}$$

The screening and quantum effects in "charge-atom" interactions. Let us introduce the different effective potentials for the interactions between neutral atoms and charged particles in plasma. We will focus on the polarization potential describing the interaction between charged particles (electrons) and neutrals. In particular, we consider the inclusion of screening and quantum-mechanical effects into the polarization potential.

As mentioned above at large distances the interaction between an isolated atom and a charged particle is given by Eq. (1). However, this potential is not appropriate for dense (nonideal) plasmas. At short distances, it becomes singular. It has to be modified if r is of the order of the extension of the atom as given by the Bohr radius a_B. According to Buckingham, a cutoff radius r_1 can be introduced as follows:

$$\Phi(r) = -\frac{e^2\alpha}{2(r^2 + r_1^2)^2} \quad , \tag{9}$$

where $r_1^4 = \alpha a_B / 2$ and we obtain the finite value of this potential $\Phi(0) = -e^2 / a_B$.

At large distances, also a modification is necessary. As known that in dense plasmas, the Coulomb interaction between charged particles is screened. We have to take into account also the screening effects in the polarization potential (R.Redmer et al, 1997):

$$\Phi(r) = -\frac{e^2\alpha}{2(r^2 + r_1^2)^2} \exp\left(-\frac{2r}{r_D}\right)\left(1 + \frac{r}{r_D}\right)^2 \quad , \tag{10}$$

where r_D is the Debye radius.

Notice that both screening and quantum effects should be taken into account in nonideal partially ionized plasma. In general case the neutral atom is polarized in an external electric field generated by charges of plasma. Thus we can consider atoms as dipoles. Let us consider a semiclassical partially ionized plasma consisting of electrons, ions and atoms. In this case the following effective potential for electron-atom interaction is obtained (T.Ramazanov et al, 2005):

$$\Phi(r)=-\frac{e^2\alpha}{2r^4\left(1-4ƛ^2/r_D^2\right)}\left(e^{-Br}(1+Br)(1-B^2ƛ^2)-e^{-Ar}(1+Ar)(1-A^2ƛ^2)\right)^2 \quad (11)$$

where

$$A^2=\frac{1}{2ƛ^2}\left(1+\sqrt{1-4ƛ^2/r_D^2}\right),$$
$$B^2=\frac{1}{2ƛ^2}\left(1-\sqrt{1-4ƛ^2/r_D^2}\right), \quad (12)$$

In (11) we take into account quantum diffraction effects only in interactions between electrons. In the limiting case $ƛ << r_D$, this potential takes the form of a well-known interaction potential (1) for isolated classical atoms and electrons.

At high densities the quantum diffraction effects must be also taken into account in considerations of atom-electron interactions. Then in this case we have the following potential for electron-atom interaction (T.Ramazanov et al, 2005):

$$\Phi(r)=-\frac{e^2\alpha}{2r^4(1-4ƛ^2/r_D^2)}\left(e^{-Br}(1+Br)-e^{-Ar}(1+Ar)\right)^2 . \quad (13)$$

Notice that at small distances between particles ($r\to 0$) this potential has the finite value $\Phi(0)=-\alpha e^2/8ƛ^2$.

Neutral and compound particles in plasma. Firstly, we will shortly discuss here about influence of atom-atom interactions in plasma. Usually the plasma interactions (Coulomb and polarization) have high intensity and long–range character and the interaction between neutral particles are weak. In order to estimate the intensity of atom-atom interactions the van der Waals type equation of state can be used:

$$(p+n_a^2 a)=\frac{n_a k_B T}{1-n_a b} \tag{14}$$

Then, the ideality criteria can be written as:

$$n_a b \ll 1, \qquad n_a a / k_B T \ll 1 \quad , \tag{15}$$

where a and b are the parameters in the van der Waals equation. These parameters can be expressed in terms of the critical temperature and density:

$$T_c = 8a/27b, \qquad n_c = (3b)^{-1} \quad . \tag{16}$$

It should be noted that interatomic interactions in plasma are important at densities close to and higher than the critical values.

The range of existence and the classification of states of nonideal plasma. Let us consider the fully ionized two–component hydrogen plasma and the corresponding existence diagram, the (n_e, T) plane in logarithmic coordinates, for such plasma (see, Figure 1). In this figure the lines corresponding to the conditions $(\gamma=1, \gamma_q=1, \xi=1)$ are also shown. These lines divide the (n_e, T) diagram into several characteristic regions.

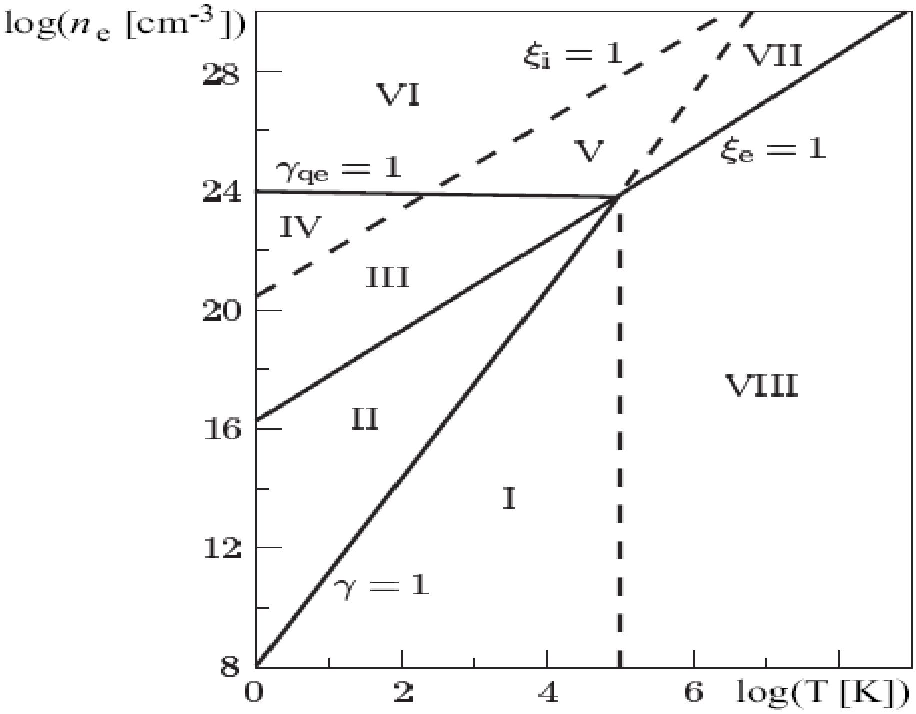

Figure 1. The range of existence for a nonideal hydrogen plasma.

Let us analyze separate regions on (n_e, T) diagram.

- Region I: $\xi_e < 1, \xi_i < 1, \gamma < 1$: a classical plasma with weak interaction of the electrons and ions.
- Region II: $\xi_e < 1, \xi_i < 1, \gamma > 1$: a classical plasma with strong interaction of the electrons and ions.
- Region III: $\xi_e > 1, \xi_i < 1, \gamma_{qe} > 1, \gamma > 1$: the electrons form a degenerate system with strong interaction while the ions form a classical system with strong interaction.
- Region IV: $\xi_e > 1, \xi_i > 1, \gamma_{qe} > 1, \gamma_{qi} > 1$: a quantum plasma with strong interaction of the electrons and ions.
- Region V: $\xi_e > 1, \xi_i < 1, \gamma_{qe} < 1, \gamma > 1$: the electrons form a degenerate system with weak interaction while the ions form a classical system with strong interaction.
- Region VI: $\xi_e > 1, \xi_i > 1, \gamma_{qe} < 1, \gamma_{qi} > 1$: the electrons are degenerate and interact weakly, the ions are degenerate and interact strongly.
- Region VII: $\xi_e > 1, \xi_i < 1, \gamma_{qe} < 1, \gamma < 1$: an electron/ion plasma with weak interactions, in which the electron component is degenerate.
- Region VIII: $\xi_e < 1, \xi_i < 1, \gamma < 1$: a classical plasma with weak interaction of the electrons and ions.

It should be noted that regions I, VII, and VIII represent gaseous plasmas at various temperatures and densities, regions V and VI correspond to a solid state. Regions III and IV also correspond to a condensed matter. Region I represents a weakly nonideal low–temperature gas discharge plasma. Region VIII corresponds to high–temperature almost ideal plasma.

Nonideal plasma in nature and its scientific and technical applications. In Figure 2 the plasma parameters realized in nature and in various technical devices are shown (W.Ebeling et al, 1976).

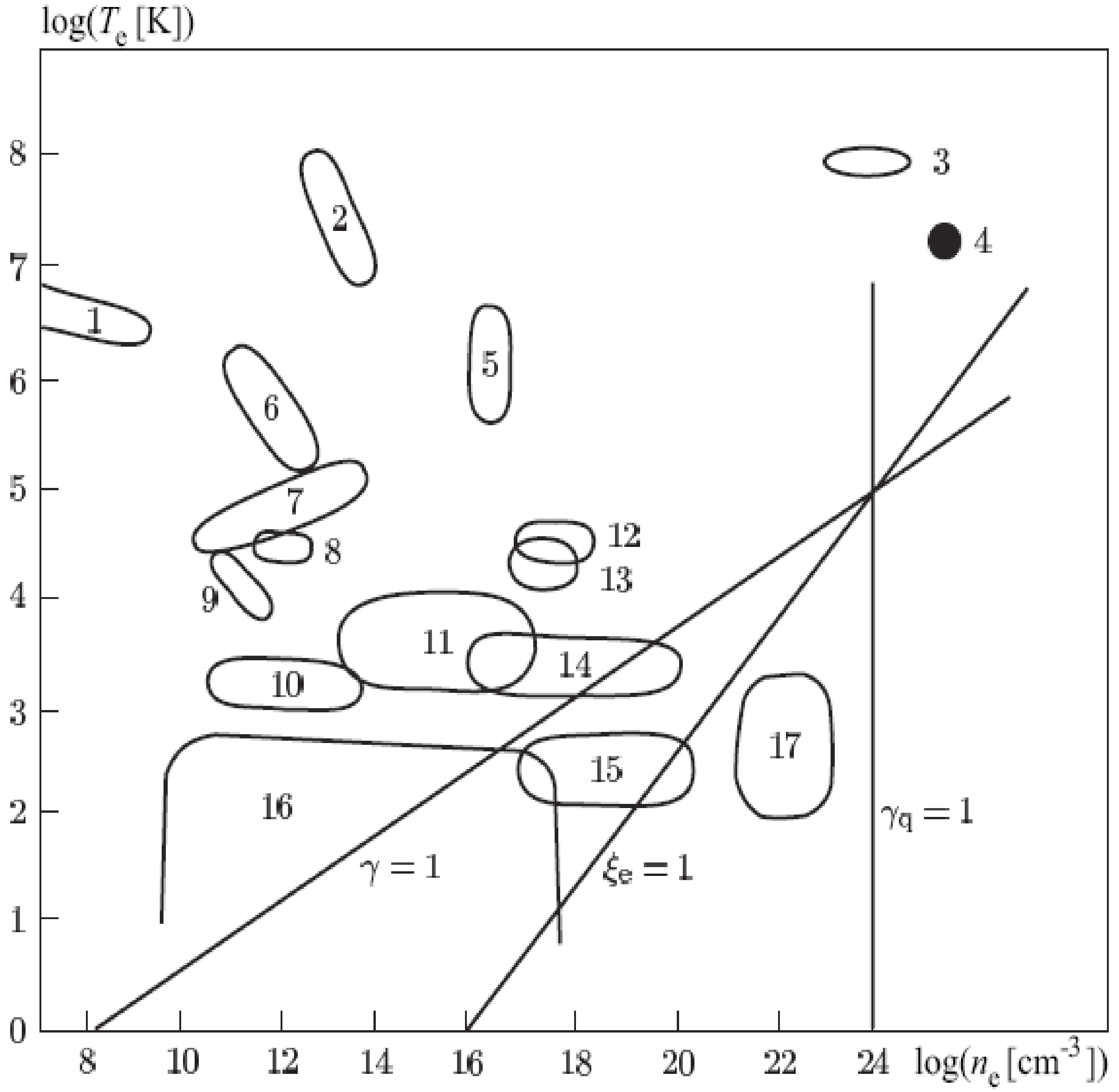

Figure 2. The plasma parameters realized in nature and in various technical devices. 1 - solar corona; 2 - tokamak; 3 - laser–induced fusion; 4 - core of Sun; 5 - Z–pinch; 6 - stellarator; 7 - gas lasers; 8 - plasmotron; 9 - chromosphere of Sun ; 10 - plasma of hydrocarbon fuel combustion products; 11 - electric arcs; 12 - cathode spot; 13 - spark; 14 - MHD generator; 15 - semiconductor plasma; 16 - metal–ammonia solutions; 17 - metals.